Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics: analysis and approaches
# Standard level
# Paper 2

2 May 2024

**Zone A** morning | **Zone B** morning | **Zone C** morning

Candidate session number

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1 hour 30 minutes

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2224 – 7110

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

**1.** [Maximum mark: 6]

In a study, the mobile phone usage of a random sample of ten students was examined on a particular day.

The length of time, $t$ hours, that the ten students used their phones for are listed below.

$$0.7 \quad 1.2 \quad 1.9 \quad 4.0 \quad 4.4 \quad 4.5 \quad 4.9 \quad 5.7 \quad 6.5 \quad 11.7$$

(a) For these data, find the

(i) median;

(ii) interquartile range. [3]

An outlier is a value that is less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$.

(b) Show that $11.7$ is an outlier. [3]

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**2.** [Maximum mark: 6]

The functions $f$ and $g$ are both defined for $-1 \leq x \leq 0$ by

$$f(x) = 1 - x^2$$

$$g(x) = \mathrm{e}^{2x}.$$

The graphs of $f$ and $g$ intersect at $x = a$ and $x = b$, where $a < b$.

(a) Find the value of $a$ and the value of $b$. [3]

(b) Find the area of the region enclosed by the graphs of $f$ and $g$. [3]

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**3.** [Maximum mark: 5]

Consider the following bivariate data set where $p, q \in \mathbb{Z}^+$.

| $x$ | 5 | 6 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $y$ | 9 | 13 | $p$ | $q$ | 21 |

The regression line of $y$ on $x$ has equation $y = 2.1875x + 0.6875$.

The regression line passes through the mean point $(\overline{x}, \overline{y})$.

(a) Given that $\overline{x} = 7$, verify that $\overline{y} = 16$. [1]

(b) Given that $q - p = 3$, find the value of $p$ and the value of $q$. [4]

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**4.** [Maximum mark: 6]

The loudness of a sound, $L$, measured in decibels, is related to its intensity, $I$ units, by $L = 10 \log_{10}(I \times 10^{12})$.

Consider two sounds, $S_1$ and $S_2$.

$S_1$ has an intensity of $10^{-6}$ units and a loudness of $60$ decibels.

$S_2$ has an intensity that is twice that of $S_1$.

(a) State the intensity of $S_2$. [1]

(b) Determine the loudness of $S_2$. [2]

The maximum loudness of thunder in a thunderstorm was measured to be $115$ decibels.

(c) Find the corresponding intensity, $I$, of the thunder. [3]

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**5.** [Maximum mark: 6]

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time $t$ seconds is given by $v(t) = 1 + \text{e}^{-t} - \text{e}^{-\sin 2t}$ for $0 \leq t \leq 2$.

(a) Find the velocity of the particle at $t = 2$. [1]

(b) Find the maximum velocity of the particle. [2]

(c) Find the acceleration of the particle at the instant it changes direction. [3]

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**6.** [Maximum mark: 5]

Consider a random variable $X$ such that $X \sim \mathrm{B}(n, 0.25)$.

Determine the least value of $n$ such that $\mathrm{P}(X \geq 1) > 0.99$.

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Do **not** write solutions on this page.

## Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

**7.** [Maximum mark: 14]

A lake contains a type of fish called carp. The lengths, $L\,\text{cm}$, of the carp can be modelled by a normal distribution with mean $45.6\,\text{cm}$ and standard deviation $4.2\,\text{cm}$.

According to this model, carp with a length between $41.4\,\text{cm}$ and $k\,\text{cm}$ lie within one standard deviation of the mean.

(a) Write down the value of $k$. [2]

(b) Find the probability that a randomly selected carp is greater than $48\,\text{cm}$ in length. [2]

(c) It is known that $99\%$ of carp in the lake have a length greater than $x\,\text{cm}$. Find the value of $x$. [2]

(d) Consider a random sample of $100$ carp from the lake.

(i) Find the expected number of carp with lengths between $40\,\text{cm}$ and $56\,\text{cm}$.

(ii) Find the probability that in this sample, exactly $95$ carp have a length between $40\,\text{cm}$ and $56\,\text{cm}$. [5]

A large sample of carp from the lake is studied. The length of each fish is measured and recorded correct to the nearest $0.1\,\text{cm}$.

(e) Find the probability that a randomly selected carp has a length recorded as $45.6\,\text{cm}$. [3]

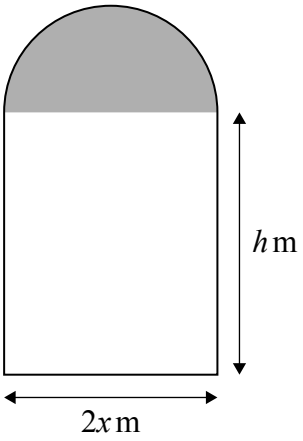
Do **not** write solutions on this page.

**8.** [Maximum mark: 15]

A window is designed in the shape of a semicircle attached to a rectangle.

The rectangular section of the window has dimensions $2x$ metres by $h$ metres.

The window consisting of its two sections is shown in the following diagram.




Let the area of the window be $A$ square metres.

(a) Write down an expression for $A$ in terms of $x$ and $h$. [2]

Let the perimeter of the window be $P$ metres.

(b) Given that $P = 10$, show that $h = \frac{1}{2}(10 - 2x - \pi x)$. [2]

The window is designed to let in the maximum amount of light.

The rectangular section of the window consists of clear glass and lets in three units of light per square metre.

The semicircular section of the window consists of tinted glass and lets in one unit of light per square metre.

(c) Show that the amount of light, $L$ units, let in by the window is given by
$L = 30x - 6x^2 - \frac{5}{2}\pi x^2$. [4]

(d) (i) Find an expression for $\frac{dL}{dx}$.

(ii) Find the value of $x$ so that the window lets in the maximum amount of light. Justify that this value of $x$ gives a maximum.

(iii) Find the value of $h$ so that the window lets in the maximum amount of light. [7]

Do **not** write solutions on this page.

**9.** [Maximum mark: 17]

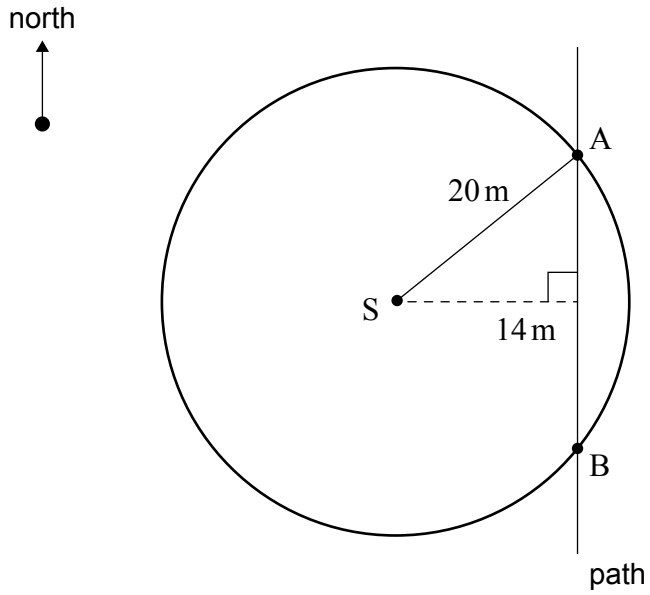
A rotating sprinkler is at a fixed point $\mathrm{S}$.

It waters all points inside and on a circle of radius $20$ metres.

Point $\mathrm{S}$ is $14$ metres from the edge of a path which runs in a north-south direction.

The edge of the path intersects the circle at points $\mathrm{A}$ and $\mathrm{B}$.

This information is shown in the following diagram.




(a) Show that $\mathrm{AB} = 28.57$, correct to four significant figures. [3]

The sprinkler rotates at a constant rate of one revolution every $16$ seconds.

(b) Show that the sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second. [1]

Let $T$ seconds be the time that $[\mathrm{AB}]$ is watered in each revolution.

(c) Find the value of $T$. [4]

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**(Question 9 continued)**

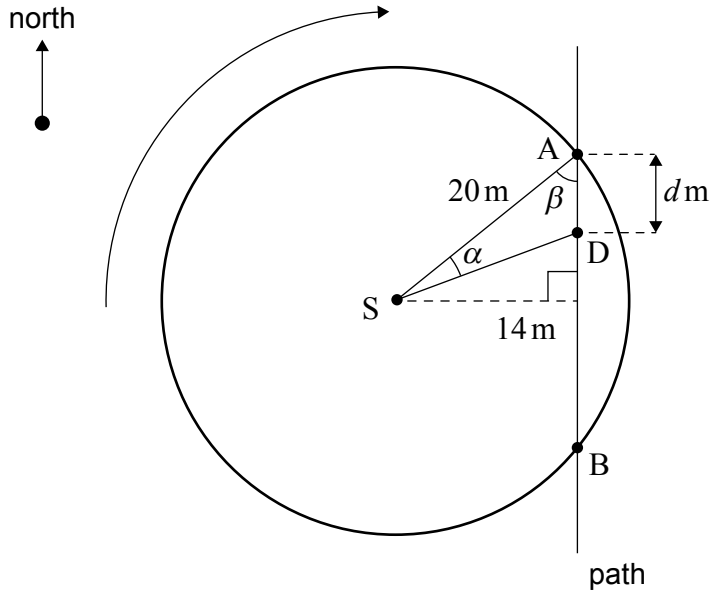
Consider one clockwise revolution of the sprinkler.

At $t = 0$, the water crosses the edge of the path at $\mathrm{A}$.

At time $t$ seconds, the water crosses the edge of the path at a movable point $\mathrm{D}$ which is a distance $d$ metres south of point $\mathrm{A}$.

Let $\alpha = \mathrm{A\hat{S}D}$ and $\beta = \mathrm{S\hat{A}B}$, where $\alpha$, $\beta$ are measured in radians.

This information is shown in the following diagram.




(d) Write down an expression for $\alpha$ in terms of $t$. [1]

It is known that $\beta = 0.7754$ radians, correct to four significant figures.

(e) By using the sine rule in $\Delta\mathrm{ASD}$, show that the distance, $d$, at time $t$, can be modelled by

$$d(t) = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}.$$ [3]

**(This question continues on the following page)**

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**(Question 9 continued)**

A turtle walks south along the edge of the path.

At time $t$ seconds, the turtle's distance, $g$ metres south of $\mathrm{A}$, can be modelled by

$$g(t) = 0.05t^2 + 1.1t + 18, \text{ where } t \geq 0.$$

(f) At $t = 0$, state how far south the turtle is from $\mathrm{A}$. [1]

Let $w$ represent the distance between the turtle and point $\mathrm{D}$ at time $t$ seconds.

(g) (i) Use the expressions for $g(t)$ and $d(t)$ to write down an expression for $w$ in terms of $t$.

(ii) Hence find when and where on the path the water first reaches the turtle. [4]